On the Path Energy of Graphs: Inequalities and Computations

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Abstract

The path matrix of a graph, defined using the maximum number of vertex-disjoint paths between vertex pairs, provides a useful framework for analyzing structural connectivity. Motivated by its spectral significance, we study the path energy of a graph. For a graph Ω with path matrix $P(\Omega)$, we derive lower and upper bounds for the path energy $PE(\Omega)$ in terms of its path eigenvalues and the number of vertices, using classical inequalities. We further show that equality holds if and only if Ω is regular. In addition, explicit expressions for the path energy are obtained for several classes of graphs, demonstrating the sharpness of the derived bounds.

Keywords: Path matrix, vertex-disjoint paths, path eigenvalues, bounds, path energy.

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